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Effects of finite width of excited states on heavy-ion sub-barrier fusion reactions

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We discuss the effects of coupling of the relative motion to nuclear collective excitations which have a finite lifetime on heavy-ion fusion reactions at energies near and below the Coulomb barrier. Both spreading and escape widths are explicitly taken into account in the exit doorway model. The coupled-channels equations are numerically solved to show that the finite resonance width always hinders fusion cross sections at sub-barrier energies irrespective of the relative importance between the spreading and the escape widths. We also show that the structure of fusion barrier distribution is smeared due to the spreading of the strength of the doorway state. [S0556-2813(98)04611-1]

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I. INTRODUCTION

Extensive experimental as well as theoretical studies during the past two decades have led to a well-established idea that cross sections for heavy-ion fusion reactions are considerably enhanced at sub-barrier energies compared with predictions of the one-dimensional potential model [1]. It has subsequently been concluded that these enhancements of fusion cross sections can be attributed to couplings between the relative motion of the colliding nuclei and several nuclear collective motions as well as transfer reactions [2]. A standard way to address the effects of channel coupling on fusion cross sections is to numerically solve the coupled-channels equations. In coupled-channels calculations involving low-lying collective excitations of medium mass nuclei, which show the large enhancements of fusion cross sections, the excited states are usually assumed to have an infinite lifetime. When the excitation energies of the collective modes exceed the threshold energy for particle emission or the typical excitation energy of incoherent modes of excitations, like giant resonances in stable nuclei, they have non-negligible lifetimes [3]. However, in stable nuclei, the excitation energy of giant resonances is so large that the effects of their excitations and thus of their width can be well described in terms of a static potential renormalization [4,5] and does not play so significant a role. Nevertheless, it is still an important issue to elucidate the effects of the finite width of excited states on fusion, since they are expected to become important in fusion reactions of weakly bound nuclei like ${}^6\text{Li}$, ${}^9\text{Be}$ [6–9] or nuclei far from the stability lines [10–14].

Some attempts to include the effects of finite width in coupled-channels calculations have been made recently by Hussein *et al.* [15,16]. They used the exit doorway model [17,18] to discuss the effects of spreading width Γ^\downarrow on fusion reactions. Instead of numerically solving the resultant coupled-channels equations as they are, they introduced the constant coupling approximation to diagonalize the coupling matrix [19] and claimed that the spreading width further enhances fusion cross sections compared with the case where the excited state has an infinite lifetime. As for the effects of escape width Γ^\uparrow , they took another model, i.e., a model which uses a dynamical polarization potential to account for

the effects of the breakup reaction. They thus showed that the escape width strongly reduces fusion cross sections.

Although the results of Refs. [15,16] are interesting, there are still some unsatisfactory aspects in their approach. First the constant coupling approximation used in Refs. [15,16] does not provide satisfactory results in heavy-ion fusion reactions where the coupling extends outside the Coulomb barrier [20–22]. Furthermore, the constant coupling approximation leads to complex values of eigenenergies as well as weight factors if one eliminates the internal degrees of freedom which couple to the doorway state. References [15,16], however, did not fully take this fact into consideration. The exact coupled-channels calculations are, therefore, urged in order to draw a definite conclusion on the effects of spreading width Γ^\downarrow on heavy-ion fusion reactions. Secondly, the effects of escape width Γ^\uparrow are not transparent in the polarization potential formalism used in Refs. [15,16]. Also it will be hard to evaluate the polarization potential for each reaction. For example, in fusion reactions of ${}^9\text{Be}$, most of the states of ${}^9\text{Be}$ which are excited during fusion will eventually decay into the $n + 2\alpha$ channel, because the separation energy of α particle is small in this nucleus. It is not so straightforward to derive a polarization potential in a reliable way for such four-body problems.

In this paper, we extend the model in Refs. [15,16] to treat both spreading and escape widths on an equal footing. This enables us to see explicitly the effects of both widths on sub-barrier fusion reactions and to easily discuss the case where both of them are present simultaneously. We then carry out exact coupled-channels calculations to investigate the effects of the finite width on heavy-ion fusion reactions. It will be shown that both spreading and escape widths reduce fusion cross sections, contrary to the conclusions in Refs. [15,16]. We also discuss the effects of the finite width on the fusion barrier distribution defined as the second derivative of the product of the fusion cross section and the center-of-mass energy, $E\sigma$ [23,24].

The paper is organized as follows. In Sec. II, we briefly review the exit doorway model and derive coupled-channels equations which account for the effects of the finite width of excited states. We consider three cases, (i) where only the spreading width exists, (ii) only the escape width exists, and

(iii) both the spreading and escape widths are present simultaneously. In Sec. III, we present numerical results of the coupled-channels calculations for these three cases and discuss the effects of finite width on fusion cross sections as well as on fusion barrier distributions. The summary is given in Sec. IV. Finally in the Appendix, a time-dependent approach to discuss the effects of escape width is presented.

II. COUPLING TO RESONANCE CHANNEL

A. Effect of spreading width

We first discuss the effects of the spreading width on subbarrier fusion reactions. To this end, we use the exit doorway model [15–18]. In this model, the relative motion between the colliding nuclei couples to many excited states only through the doorway state, i.e., the collective state. We assume the following Hamiltonian for the fusing system:

$$H = -\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{J(J+1)\hbar^2}{2\mu r^2} + V_N(r) + \frac{Z_P Z_T e^2}{r} + H_{\text{int}}(\xi) + V_{\text{coup}}(r, \xi), \quad (1)$$

where r is the coordinate of the relative motion between the projectile and the target, and μ is the reduced mass. V_N is the bare nuclear potential, Z_P and Z_T are the atomic numbers of the projectile and the target, respectively. H_{int} describes the intrinsic excitations in one of the colliding nuclei, and V_{coup} is the coupling between these excitations (generically denoted by ξ) and the relative motion. In writing Eq. (1), we used the no-Coriolis approximation and replaced the angular momentum operator for the relative motion by the total angular momentum J [25,26]. Following Refs. [15,16,18], we assume that the intrinsic Hamiltonian H_{int} and the coupling Hamiltonian V_{coup} are given, respectively, by

$$H_{\text{int}} = |d\rangle E_d \langle d| + \sum_j |j\rangle e_j \langle j| + \sum_j [|j\rangle \Delta_j \langle d| + |d\rangle \Delta_j^* \langle j|], \quad (2)$$

$$V_{\text{coup}} = f(r)(|0\rangle \langle d| + |d\rangle \langle 0|), \quad (3)$$

where $|0\rangle$, $|d\rangle$, and $|j\rangle$ denote the ground state, the doorway state, and the other intrinsic states, respectively. E_d and e_j are the energy of the doorway state and that of the state $|j\rangle$, respectively. In our example in this subsection where we discuss the effects of spreading width, $|j\rangle$ are uncorrelated $1p1h$ states or more complicated many particle many hole states within the same nucleus. The former yields the Landau damping, while the latter the spreading width [3]. In Eqs. (2) and (3), Δ and $f(r)$ are the coupling strength between the doorway state and $|j\rangle$, and between the doorway state and the ground state, respectively. Inherently, the former is independent of r . For simplicity we have assumed that the doorway state $|d\rangle$ linearly couples to the ground state $|0\rangle$. The extension to the case where there exist higher-order couplings [30] is straightforward.

The intrinsic Hamiltonian H_{int} can be diagonalized by introducing the normal modes $|\varphi_i\rangle$ by

$$H_{\text{int}} = \sum_i |\varphi_i\rangle E_i \langle \varphi_i|, \quad (4)$$

$$|\varphi_i\rangle = \alpha_i |d\rangle + \sum_j \beta_{ij} |j\rangle. \quad (5)$$

When the states $|j\rangle$ are distributed with equal energy spacing from $-\infty$ to ∞ , and the coupling strengths Δ_j are independent of j , i.e.,

$$e_j = jD \quad (j=0, \pm 1, \pm 2, \dots), \quad (6)$$

$$\Delta_j = \kappa, \quad (7)$$

E_i and α_i in Eqs. (4) and (5) are given by

$$E_i = E_d + \frac{\pi \kappa^2}{D} \cot \frac{\pi E_i}{D}, \quad (8)$$

$$|\alpha_i|^2 = \frac{1}{2\pi/D} \frac{\Gamma^\perp}{(E_i - E_d)^2 + \Gamma^{\perp 2}/4}, \quad (9)$$

respectively (the Breit-Wigner distribution) [17]. Here Γ^\perp is the spreading width, including the Landau width, and is defined by $\Gamma^\perp = 2\pi\kappa^2/D$. In obtaining Eq. (9), we assumed that the coupling strength κ is much larger than the energy spacing D and neglected a term of the order of D^2/κ^2 in the denominator.

Expanding the total wave function with the eigenstates $|\varphi_i\rangle$ as

$$\Psi(r, \xi) = u_0(r)|0\rangle + \sum_i u_i(r)|\varphi_i\rangle, \quad (10)$$

the coupled-channels equations read

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{J(J+1)\hbar^2}{2\mu r^2} + V_N(r) + \frac{Z_P Z_T e^2}{r} - E \right] u_0(r) + \sum_j \alpha_j f(r) u_j(r) = 0, \quad (11)$$

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{J(J+1)\hbar^2}{2\mu r^2} + V_N(r) + \frac{Z_P Z_T e^2}{r} - E + E_j \right] u_j(r) + \alpha_j^* f(r) u_0(r) = 0. \quad (12)$$

These equations may be solved by imposing the incoming wave boundary condition inside the Coulomb barrier, i.e.,

$$u_i(r) = T_i \exp \left(-i \int_{r_{\text{abs}}}^r k_i(r') dr' \right) \quad r \leq r_{\text{abs}}, \quad (13)$$

$$= H_j^{(-)}(k_i r) \delta_{i,0} + R_i H_j^{(+)}(k_i r) \quad r \rightarrow \infty, \quad (14)$$

where $k_i(r)$ is the local wavelength for the i th channel, and r_{abs} is the absorption radius where the incoming wave bound-

ary condition is imposed. $H_j^{(-)}$ and $H_j^{(+)}$ are the incoming and the outgoing Coulomb waves, respectively. The fusion cross section is then obtained as

$$\sigma(E) = \frac{\pi}{k^2} \sum_J (2J+1) \left(\frac{k_0(r_{\text{abs}})}{k} |T_0|^2 + \sum_i \frac{k_i(r_{\text{abs}})}{k} |T_i|^2 \right), \quad (15)$$

where $k = k_0(\infty)$ is the wavelength of the entrance channel at $r \rightarrow \infty$. Note that in contrast to the works by Hussein *et al.* which eliminate the intrinsic degree of freedom $|j\rangle$ and introduce the complex Q value [15,16], we treat it explicitly in the coupled-channels calculations.

B. Effect of escape width

We next consider the effects of escape width. To this end, we use the same model as is used in the previous section, i.e., the exit doorway model. In this case, the states $|j\rangle$ represent particle continuum states. One may imagine the box normalization of these states in order to match with the discretization of the continuum states $|j\rangle$. Although it is not obvious whether the exit doorway model, and especially the Breit-Wigner assumptions given by Eqs. (6) and (7), are appropriate to describe the effects of an escape width, our results do not depend so much upon the nature of the strength distribution as we will show in the next section. More appropriate way taking into account the irreversible coupling to the open channel may be provided by, e.g., the continuum random-phase-approximation (RPA) formalism [27–29]. The coupled-channels equations to be solved are exactly the same as Eqs. (11) and (12). The difference appears at the final stage. We define the complete fusion as such a process, where the whole projectile is absorbed by the whole target without emitting any particle prior to the fusion. Therefore, the particle continuum states $|j\rangle$ have to be excluded from the final states in obtaining the cross section for the complete fusion. Since the probability to find a state $|j\rangle$ in the state for the i th normal mode $|\varphi_i\rangle$ is given by $|\beta_{ij}|^2$, the complete fusion cross section is given by

$$\sigma(E) = \frac{\pi}{k^2} \sum_J (2J+1) \times \left(\frac{k_0(r_{\text{abs}})}{k} |T_0|^2 + \sum_i |\alpha_i|^2 \frac{k_i(r_{\text{abs}})}{k} |T_i|^2 \right). \quad (16)$$

In deriving Eq. (16), we used the normalization condition of the normal states, i.e., $|\alpha_i|^2 = 1 - \sum_j |\beta_{ij}|^2$.

Another approach to fusion reactions in the presence of a breakup channel is to evaluate the loss of flux during fusion [11–13]. The relation between such an approach and the present formalism is given in the Appendix by using a time-dependent theory.

C. Interplay between spreading and escape widths

Lastly we consider the case where both spreading and escape widths are present simultaneously and interplay with each other. In this case, the intrinsic Hamiltonian is given by

$$\begin{aligned} H_{\text{int}} = & |d\rangle E_d \langle d| + \sum_j |j^\uparrow\rangle e_j^\uparrow \langle j^\uparrow| + \sum_j |j^\downarrow\rangle e_j^\downarrow \langle j^\downarrow| \\ & + \sum_j [|j^\uparrow\rangle \Delta_j^\uparrow \langle d| + |d\rangle \Delta_j^{\uparrow*} \langle j^\uparrow|] \\ & + \sum_j [|j^\downarrow\rangle \Delta_j^\downarrow \langle d| + |d\rangle \Delta_j^{\downarrow*} \langle j^\downarrow|], \end{aligned} \quad (17)$$

where $|j^\uparrow\rangle$ and $|j^\downarrow\rangle$ represent particle continuum states and complicated particle hole bound states, respectively. We diagonalize this Hamiltonian by introducing the normal states defined by

$$|\varphi_i\rangle = \alpha_i |d\rangle + \sum_j \beta_{ij}^\uparrow |j^\uparrow\rangle + \sum_j \beta_{ij}^\downarrow |j^\downarrow\rangle. \quad (18)$$

As in Sec. II A, if we assume a uniformly spaced sequence of energies e_j^\uparrow and e_j^\downarrow and state independent coupling strengths Δ_j^\uparrow and Δ_j^\downarrow , the eigenvalues E_i and the doorway amplitude α_i are given by

$$E_i = E_d + \frac{\pi(\kappa^{\uparrow 2} + \kappa^{\downarrow 2})}{D} \cot \frac{\pi E_i}{D}, \quad (19)$$

$$|\alpha_i|^2 = \frac{1}{2\pi/D} \frac{\Gamma^\uparrow + \Gamma^\downarrow}{(E_i - E_d)^2 + (\Gamma^\uparrow + \Gamma^\downarrow)^2/4}, \quad (20)$$

respectively. We have assumed that the energy spacing of the particle continuum states $|j^\uparrow\rangle$ is the same as that of the particle-hole bound states $|j^\downarrow\rangle$. Here Γ^\uparrow and Γ^\downarrow are escape and spreading widths given by $2\pi\kappa^{\uparrow 2}/D$ and $2\pi\kappa^{\downarrow 2}/D$, respectively. Excluding the particle continuum states $|j^\uparrow\rangle$ from the final states, the complete fusion cross section is given by

$$\begin{aligned} \sigma(E) = & \frac{\pi}{k^2} \sum_J (2J+1) \\ & \times \left[\frac{k_0(r_{\text{abs}})}{k} |T_0|^2 + \sum_i \left(|\alpha_i|^2 + \frac{\Gamma^\downarrow}{\Gamma^\uparrow + \Gamma^\downarrow} \right) \right. \\ & \left. \times \frac{k_i(r_{\text{abs}})}{k} |T_i|^2 \right]. \end{aligned} \quad (21)$$

In deriving Eq. (21), we used the identity $\sum_j |\beta_{ij}^\downarrow|^2 = \Gamma^\downarrow/(\Gamma^\uparrow + \Gamma^\downarrow)$. The complete fusion cross section is thus intimately related to the ratio $\Gamma^\downarrow/(\Gamma^\uparrow + \Gamma^\downarrow)$.

III. EFFECTS OF FINITE WIDTH ON FUSION CROSS SECTIONS AND BARRIER DISTRIBUTIONS

We now present the results of our calculations of fusion cross sections and fusion barrier distributions. We consider fusion reactions between ^{16}O and ^{144}Sm in the presence of low-lying octupole phonon excitations in the latter nucleus. We artificially set its excitation energy E_d to be 2 MeV and assume that it has a hypothetical total width of 1 MeV. Our conclusions do not depend so much on the particular choice

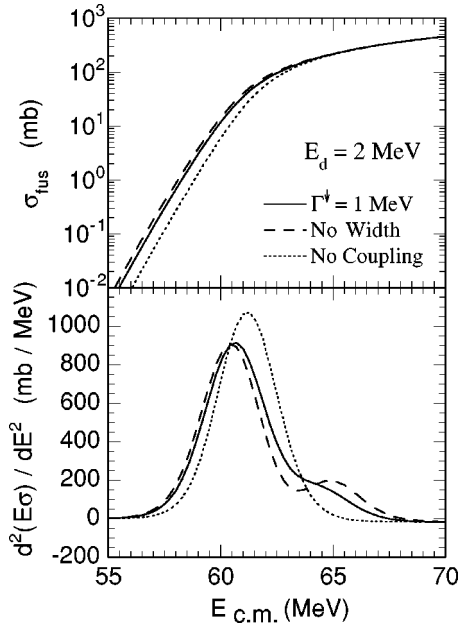


FIG. 1. Effects of the spreading width on the fusion cross section (the upper panel) and fusion barrier distribution (the lower panel). The dotted line is for the case of no coupling. The solid line takes into account the effects of coupling of the relative motion to a doorway state at 2 MeV with the width 1 MeV, while the dashed line assumes that the doorway state has an infinite lifetime.

for these parameters. We use the collective model for the coupling form factor $f(r)$, i.e.,

$$f(r) = \frac{\beta_3}{\sqrt{4\pi}} \left(-R_T \frac{dV_N}{dr} + \frac{3}{2\lambda+1} Z_p Z_T \frac{R_T^\lambda}{r^{\lambda+1}} \right), \quad (22)$$

where $\lambda=3$ is the multipolarity of the excitation and R_T is the radius of ^{144}Sm . β_3 is the deformation parameter of the phonon excitation, which was chosen to be 0.205 with the target radius of $R_T=1.06A^{1/3}$ fm. We used the same nuclear potential V_N as that in Refs. [5,30], i.e., a Woods-Saxon potential whose depth, range parameter, and surface diffuseness are $V=105.1$ MeV, $r_0=1.1$ fm, and $a=0.75$ fm, respectively. In the actual calculations, we introduced a cutoff energy and a finite energy spacing for the excited states by considering the normal states between E_d-1 and E_d+1 MeV with the energy spacing D of 0.2 MeV. We have checked that our conclusions do not qualitatively alter when the cutoff energy is taken to be larger and/or the energy spacing smaller.

We first discuss the effects of spreading width. The upper panel of Fig. 1 shows the fusion cross section of this system. The solid line was obtained by numerically solving the coupled-channels equations Eqs. (11) and (12) with $\Gamma^\downarrow=1$ MeV, while the dashed line is the result when the doorway state has an infinite lifetime. The figure also contains the result for the no coupling case (the dotted line) for comparison. One can see that the spreading width slightly reduces the fusion cross section, though it is still enhanced compared with the case of no coupling. Our result contradicts the conclusion in Refs. [15,16] where it was claimed that the spreading width enhances the fusion cross section. This discrepancy could be associated with the constant coupling model

and/or the incorrect treatments of complex potentials and weight factors in Refs. [15,16].

The fact that the spreading width reduces the enhancement of fusion cross section can be understood in the following way. After eliminating the intrinsic states $|j\rangle$, the coupled-channels problem given by Eqs. (11) and (12) reduces to the two-channel problem with the coupling matrix [15,16] (see, also, the Appendix)

$$\begin{pmatrix} 0 & f(r) \\ f(r) & E_d - i \frac{\Gamma^\downarrow}{2} \end{pmatrix}. \quad (23)$$

If we diagonalize this coupling matrix with a biorthogonal basis, the lower potential barrier is given by [15,16]

$$V_-(r) = V_N(r) + \frac{Z_p Z_T e^2}{r} + \frac{1}{2} \left(E_d - i \frac{\Gamma^\downarrow}{2} - \sqrt{E_d^2 - \frac{\Gamma^{\downarrow 2}}{4} + 4f(r)^2 - iE_d \Gamma^\downarrow} \right). \quad (24)$$

The real part of the lower barrier thus always increases when the width Γ is nonzero, leading to smaller penetrabilities at energies below the barrier. When the doorway energy E_d is much larger than the coupling form factor $f(r)$, Eq. (24) is transformed to

$$V_-(r) = V_N(r) + \frac{Z_p Z_T e^2}{r} - f(r)^2 \frac{E_d + i(\Gamma^\downarrow/2)}{E_d^2 + (\Gamma^{\downarrow 2}/4)}, \quad (25)$$

which is identical to the adiabatic barrier derived from Eq. (B21) in Ref. [18].

One might expect that the fusion cross section is enhanced if the spreading width is finite, since the doorway state then couples to the environmental background whose energy is lower than that of the doorway state itself and thus the effective excitation energy of the doorway state becomes lower. However, this intuitive picture does not hold if there is non-negligible coupling $f(r)$ around the barrier position, which modifies the potential barrier according to Eq. (24). Since the fusion cross section is much more sensitive to the barrier height at energies well below the Coulomb barrier than the energy transfer which takes place before the projectile hits the Coulomb barrier, the net effects of the spreading width cause the hindrance of the fusion cross section. A similar importance of the potential renormalization concerning the effects of transfer reaction with positive Q value on sub-barrier fusion reactions has been pointed out in Ref. [31].

This contrasts to the situation where the doorway state couples to a small number of intrinsic states $|j\rangle$. In such cases, the coupling could further enhance the fusion cross section, as is usually the case in double phonon couplings [32]. When the doorway state couples to a large number of surrounding states, as is discussed in this paper, the relaxation time becomes very short and consequently the couplings begin to reveal a dissipative character [18]. The hindrance of fusion cross sections due to the spreading width thus resembles the dissipative quantum tunneling, which has

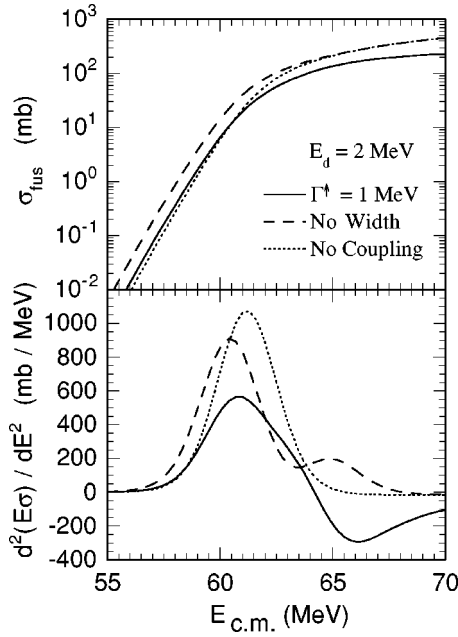


FIG. 2. Same as Fig. 1, but for the escape width.

been an extremely popular subject during the past two decades in many fields of physics and chemistry [33,34].

The above conclusions have been reached by assuming the Breit-Wigner distribution given by Eq. (9) for the non-collective states $|j\rangle$. In order to test the sensitivity to the property of the distribution, we repeated the calculations by assuming the Lorentzian distribution (not shown) and obtained the similar conclusions concerning the role of the finite resonance width in heavy-ion fusion reactions. The difference is negligible especially when the doorway energy E_d is larger than the width Γ .

The lower panel of Fig. 1 shows fusion barrier distributions for this system. This quantity is defined as the second derivative of $E\sigma$ with respect to the energy E [23], and has been experimentally shown to be very sensitive to the nuclear structure of the colliding nuclei [24]. We used the three-point difference formula with an energy spacing of 1.8 MeV to obtain the second derivative from the fusion cross sections [24]. The meaning of each line is the same as that in the upper panel. When there exists no coupling between the ground and the doorway states, the barrier distribution has only a single peak, corresponding to a single potential barrier, i.e., the bare potential barrier. If the coupling is turned on, the single barrier splits into two and the barrier distribution has two peaks (the dashed line). This double-peaked structure is somewhat smeared when the width of the doorway state is finite, since it redistributes the strength of the doorway state (the solid line). As a consequence, the higher peak of the fusion barrier distribution becomes less apparent.

We next discuss the effects of escape width. The solid line in Fig. 2 was obtained by setting $\Gamma^\dagger = 1$ MeV and $\Gamma^\downarrow = 0$ MeV, and using Eq. (16). The dotted and the dashed lines are the same as in Fig. 1. As will be discussed in the Appendix, the escape width is intimately related to a loss of flux due to the breakup reaction, and strongly hinders the fusion cross section over a wide range of bombarding energies. The fusion cross section is smaller even than that in the absence of the channel couplings at energies above the Cou-

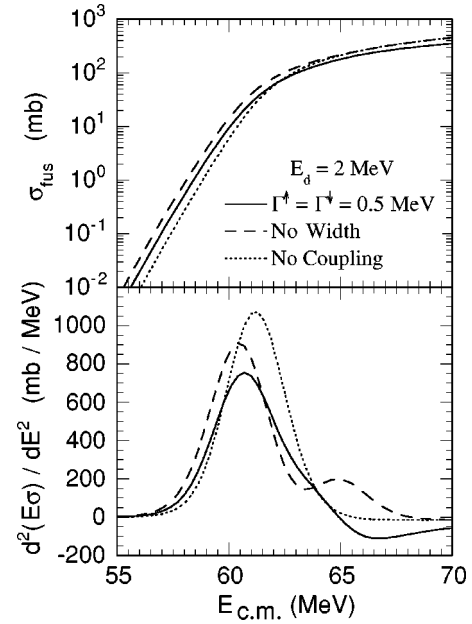


FIG. 3. Same as Fig. 1, but in the simultaneous presence of spreading and escape widths. Both widths are assumed to be 0.5 MeV.

lomb barrier. The escape width also lowers the height of the main peak of the fusion barrier distribution and at the same time broadens the fusion barrier distribution.

When there exist both spreading and escape widths simultaneously, one expects to have a situation intermediate between that depicted in Figs. 1 and 2. In order to demonstrate this, Fig. 3 shows the result when both widths are set to 0.5 MeV. Since both spreading and escape widths always reduce the fusion cross section, they are smaller than those in the infinite lifetime case over the entire energy range shown in Fig. 3. One finds that the degree of hindrance is intermediate between Figs. 1 and 2, as expected.

IV. SUMMARY

We have derived coupled-channel equations which take into account the effects of the finite width of an excited state. This formalism treats the spreading and the escape widths on an equal footing, and thus enables one to discuss an interplay between them. To this end, we used the exit doorway model. Numerical solutions of the coupled-channel equations showed that both widths hinder the fusion cross section. The degree of hindrance is moderate when the spreading width dominates the total width, while the fusion cross section is considerably reduced in the opposite case, i.e., when the escape width dominates the total width. We also investigated the effects of finite width on the fusion barrier distribution. We demonstrated that the spreading width smears the structure of the fusion barrier distribution and also that the escape width lowers the height of the main peak of the fusion barrier distribution. These considerations are important when one analyzes high precision measurements of the fusion reactions of fragile nuclei like ${}^6\text{Li}$ or ${}^9\text{Be}$, which have been undertaken recently, or when one discusses fusion reactions of unstable nuclei. We will report analyses of these experimental data in a separate paper.

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APPENDIX: TIME-DEPENDENT APPROACH OF THE EFFECT OF ESCAPE WIDTH

In this appendix, we discuss the relation between the coupled-channels formalism discussed in Sec. II B and an approach using the flux loss during fusion due to a breakup reaction. To this end, we use a time-dependent approach. Assuming that the total wave function at time t is given by

$$|\Psi(t)\rangle = a_0(t)|0\rangle + \sum_i a_i(t)e^{-iE_it/\hbar}|\varphi_i\rangle, \quad (\text{A1})$$

the time-dependent coupled equations which correspond to Eqs. (11) and (12) read [35,36]

$$\dot{a}_0(t) = \sum_i \frac{1}{i\hbar} \alpha_i f(r(t)) e^{-iE_it/\hbar} a_i(t), \quad (\text{A2})$$

$$\dot{a}_i(t) = \frac{1}{i\hbar} \alpha_i^* f(r(t)) e^{iE_it/\hbar} a_0(t). \quad (\text{A3})$$

If we assume the Breit-Wigner distribution Eq. (9) for the incoherent states $|j\rangle$, these two equations can be combined to give [35,36]

$$\begin{aligned} \dot{a}_0(t) = & \\ & - \frac{f(r(t))}{\hbar^2} \int_{-\infty}^t dt' e^{-i[E_d - i(\Gamma/2)](t-t')/\hbar} a_0(t') f(r(t')). \end{aligned} \quad (\text{A4})$$

In deriving this equation, we have assumed that the energy spacing of the states $|j\rangle$ is small enough and replaced the summation over the normal states $|\varphi_i\rangle$ with an integration over the energy of these states E_i .

Here we introduce the doorway amplitude by

$$a_d(t) = \frac{1}{i\hbar} \int_{-\infty}^t dt' e^{-i[E_d - i(\Gamma/2)](t-t')/\hbar} a_0(t') f(r(t')). \quad (\text{A5})$$

Equation (A4) can then be written in a form of two coupled equations as

$$\dot{a}_d(t) = \frac{1}{i\hbar} \left(E_d - i\frac{\Gamma}{2} \right) a_d(t) + \frac{1}{i\hbar} f(r(t)) a_0(t), \quad (\text{A6})$$

$$\dot{a}_0(t) = \frac{1}{i\hbar} a_d(t) f(r(t)). \quad (\text{A7})$$

These equations provide a two-level problem with the coupling matrix given by Eq. (23) [15,16].

When the escape width dominates the total width, i.e., $\Gamma \sim \Gamma^\dagger$, the survival probability of the system at time t is given by

$$P_s(t) = |\langle 0 | \Psi(t) \rangle|^2 + |\langle d | \Psi(t) \rangle|^2. \quad (\text{A8})$$

Noticing that $\langle d | \Psi(t) \rangle$ is nothing but the doorway amplitude $a_d(t)$, one can easily show that the time dependence of the survival probability is given by

$$\frac{d}{dt} P_s(t) = - \frac{\Gamma^\dagger}{\hbar} |a_d(t)|^2. \quad (\text{A9})$$

The survival probability $P_s(t)$ is thus a decreasing function of time t , and $1 - P_s(t)$ represents the probability of the flux loss caused by the particle emission.

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